ELECTRICAL SIMULATION OF HEATING AT A NONSTATIONARY FRICTION CONTACT

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The temperature at a nonstationary sliding friction contact is determined with the aid of a Volterra-Fredholm type integral equation of the second kind. In order to solve this equation on a standard analog computer transformations necessary to permit the use of the method of successive approximations are introduced.

In evaluating the wear resistance, antiseizing properties, and surface strength of rubbing parts it is very important to know the temperature distribution in the parts and, in particular, the temperature at the sliding contact itself.

The discrete elements of the actual contact area are the sources of friction heat. If for the purposes of simplification these discrete elements are replaced with continuous surface layers and these boundary layers are treated as a single heat-generating layer, then the thermal problem can be represented in integral form with a particular solution in the form of single and double-layer potentials. This representation makes it possible to use not only the methods of mathematical physics but also mathematical simulation on an analog computer.

Let the rubbing parts consist of a slider bearing and a rotating shaft. This system is formalized as two cylindrical half-spaces in contact along the generators. The heat source is the contact region s created by very thin surface layers of the bearing and the neck of the shaft in the area of contact. The contact region s moves at constant speed. The contact is nonstationary. This nonstationarity is due either to the nonstationarity of the relative slip velocity u or to the nonstationarity of the shear stress τ or to the nonstationarity of u and τ together [1].

The thermophysical characteristics of the rubbing materials are assumed to be different, i.e., the thermal conductivities of the bearing and shaft materials are not equal ($\lambda_1 \neq \lambda_2$), just as their specific heats ($c_1 \neq c_2$) and specific weights ($\gamma_1 \neq \gamma_2$) are different. There are no heat losses to the ambient medium, i.e., the noncontacting surfaces are adiabatic.

It is required to determine the temperature T directly at the sliding contact and the temperature distribution over the rubbing parts.

We will select a rectangular coordinate system with origin inside the contact region s. The XOY plane coincides with the common fictitious plane boundary of the rubbing bodies.

Within each of the parts the axes z_1, z_2 have the same direction as the inward normals to the boundary surface between the neck of the shaft and the bearing.

With the above assumption concerning the presence of a continuous heat-generating layer between the rubbing parts, the following relations must be satisfied [2]:

1) within the region s the temperatures of the parts are equal, i.e.,

$$T_1 = T_2, \tag{1}$$

2) the sum of the specific heat flows into the bearing q_1 and the shaft q_2 generated at any point of the heat-generating layer is equal to the specific friction power at that point, i.e.,

$$q_1 + q_2 = \frac{u\tau}{j} = f. \tag{2}$$

In the absence of internal heat sources and provided that the thermophysical characteristics of the materials do not depend on temperature and pressure, the heat conduction equation describing the temperature distribution in the bearing and shaft has the form

$$\frac{1}{a_j}\frac{\partial T_j}{\partial t} = \nabla^2 T_j \quad (j = 1,\ 2), \tag{3}$$

where

$$a_j = \frac{\lambda_j}{c_j \gamma_j},\quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

The boundary condition at the initial instant of time t = 0 is written thus:

$$T_j = T_j(x,\ y,\ z).$$

Since in the contact region s Eqs. (1) and (2) hold, in the coordinate system adopted the boundary conditions for that region are expressed as follows:

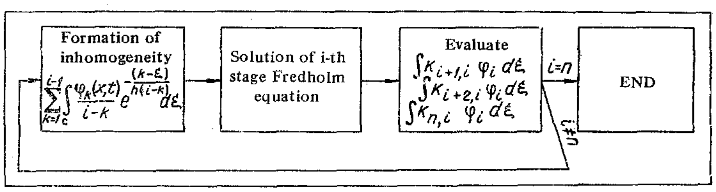

Fig. 1. Block diagram of program for solving integral equation of the second kind.

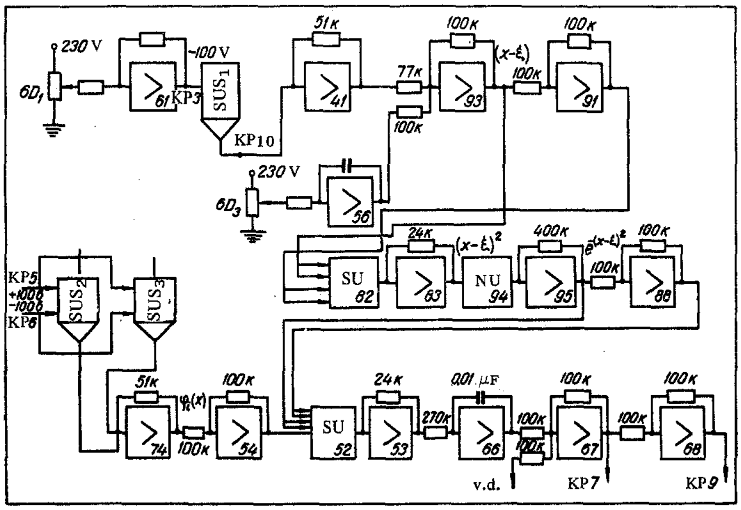

Fig. 2. Block diagram of analog simulation of integral equation of the second kind.

$$T_1(x,\ y+0;\ t)=T_2(x,\ y+0;\ t), \tag{4}$$

$$\lambda_1\frac{\partial T_1}{\partial z_1}(x,\ y+0;\ t)+\lambda_2\frac{\partial T_2}{\partial z_2}(x,\ y+0;\ t)= =-\frac{u\tau}{j}. \tag{5}$$

The problem will be solved for the case when, apart from time, the temperature T_i depends only on two coordinates (x, z). This is because in long cylindrical regions (bearings) the three-dimensional thermal processes can be reduced to plane processes without serious loss of accuracy [3]. Thus for the plane case Eq. (5) becomes

$$\lambda_1\frac{\partial T_1}{\partial z_1}(x,\ +0;\ t)+\lambda_2\frac{\partial T_2}{\partial z_2}(x,\ +0;\ t)=-\frac{u\tau}{j}.$$

Since the bearing and shaft are usually made of different materials, we must consider the region s as a region of contact between two heterogeneous media in and between which heat is propagated. In accordance with [3], at such a boundary there is a thermal discontinuity. In order to ensure the continuity of the thermal layer in the contact region, we make it a double region and dispose along the boundaries a single thermal layer on one side and a double thermal layer on the other.

In this formulation of the problem the determination of the temperature $T_1(x, z_1)$ and $T_2(x, z_2)$ reduces to the corresponding problem of potential theory which, together with conditions (3) and (4), makes it possible to represent the unknown temperatures in the form of single and double-layer potentials:

$$T_1(x,\ z_1)= =\int_0^t d\tau\int_l\frac{\varphi_1(\xi,\ \tau)}{2\pi(t-\tau)}\exp\left[-\frac{r_1^2}{4aj(t-\tau)}\right]d\xi,$$

$$T_2(x,\ z_2)=\int_0^t d\tau\int_l\frac{\varphi_2(\xi,\ \tau)}{2\pi(t-\tau)}\times$$

$$\times\frac{\partial}{\partial n}\exp\left[-\frac{r_2^2}{4aj(t-\tau)}\right]d\xi.$$

Hence we obtain the unknown value of the temperature at selected points of the region s from a Volterra-Fredholm type integral equation of the second kind:

$$f(x,\ t)=\varphi_i(x,\ t)+\int_0^t d\tau\int_l\frac{\varphi_i(x,\ t,\ \xi,\ \tau)}{t-\tau}\times$$
$$\times\left\{\frac{\lambda_2}{\lambda_1}\exp\left[-\mathrm{Pe}_1\frac{(x-\xi)^2}{t-\tau}\right]+\right.$$
$$\left.+\frac{\partial}{\partial n}\exp\left[-\mathrm{Pe}_2\frac{(x-\xi)^2}{t-\tau}\right]\right\}d\xi+$$
$$+\frac{\mathrm{Pe}}{\pi t}\int_{-\infty}^{+\infty}\int\varphi(\xi,\ \eta)\times$$
$$\times\exp\left[-\mathrm{Pe}\frac{r_i^2}{t}\right]d\xi\,d\eta. \tag{6}$$

Here, ξ is the coordinate of the variable point in the region; $\varphi_i(\xi, t)$ the unknown single and double-layer potential densities; $r_i = (x-\xi)^2 + z_i^2)^{1/2}$ the distance from some point (x, z_i) to a point (ξ) in the region s. The last term on the right side of Eq. (6) characterizes the initial condition [3].

Since normals to the planes bounding the bearing and the shaft are parallel and have the same direction as the z-axes, using (6) and the known properties of the normal derivative of single and double layers, from boundary condition (5) we obtain

$$\varphi_1(x,\ \tau)+\varphi_2(x,\ \tau)=-\frac{u\tau}{j}. \tag{6a}$$

Hence it follows that the potential densities $\varphi_i(x, \tau)$ are nothing other than the rates of heat flow into each of the rubbing parts in the region s from the moment of action τ.

These rates are easily found using boundary conditions (1) and Eqs. (6) and (6a):

$$\varphi_i(x) = \frac{\lambda_i}{\lambda_1 + \lambda_2} \frac{u\tau}{j} (i = 1,\ 2).$$

For simplicity, the problem was first solved on the assumption that $\lambda_1 = \lambda_2$, $a_1 = a_2$, and that the Peclet numbers for each part were constant and equal $Pe_1 = Pe_2 = 1$. Moreover, it was assumed that the last term on the right side of Eq. (6) was equal to zero. This assumption can be made without loss of generality since in the event of a nonzero initial condition the problem can be transformed to the above-mentioned form (6).

As a result we obtain the equation

$$f(x,\ t) = \varphi(x,\ t) + \int_0^t d\tau \int_l \frac{\varphi(x,\ t,\ \xi,\ \tau)}{t-\tau} \exp\left[-\frac{(x-\xi)^2}{t-\tau}\right] d\xi. \quad (7)$$

The function $f(x, t)$ is the heat flux in the contact region. This function is known; it is proportional to the friction power. The function $\varphi(x, t)$ is the thermal layer so distributed in the contact region that the boundary condition is satisfied; $\varphi(\xi, \tau)$ is the unknown density of the thermal layer, which makes it possible to find the temperature distribution at any point x, z at any time t, i.e., the density operating at time τ at the point (ξ, η) when this action with respect to time t is considered at points (x, z);

$$T(r,\ t) = \int_0^t d\tau \int_l \varphi(x,\ \xi,\ t,\ \tau) r(x,\ \xi) d\xi.$$

The method used to solve the given Volterra-Fredholm equation of the second kind may be divided into two distinct stages:

1) solution of a Fredholm equation of the second kind written for some moment of time t_j;

2) determination of the auxiliary quantities used as inhomogeneity function (see Fig. 1).

The time interval (0, t) is divided into n equal parts with step h = t/n.

The first integral in Eq. (7) is a finite sum calculated, for example, in accordance with the rectangle formula. As a result we obtain

$$f_i(x) = \varphi_i(x) + h \sum_{k=1}^{i} \int_l \frac{\varphi_k(x,\ \xi)}{h(i-k)} \exp\left[-\frac{(x-\xi)^2}{h(i-k)}\right] d\xi. \quad (8)$$

This equation may be transformed as follows:

$$f_i(x) - \sum_{k=1}^{i-1} \int_l \frac{\varphi_k(x,\ \xi)}{i-k} \exp\left[-\frac{(x-\xi)^2}{h(i-k)}\right] d\xi = \varphi_i(x) + \int_l \varphi_i(x,\ \xi) \exp\left[-\frac{(x-\xi)^2}{h}\right] d\xi. \quad (9)$$

In Eqs. (8) and (9)

$$f_i(x) = f(x,\ t_i),$$
$$\varphi_i(x) = \varphi(x,\ t_i),$$
$$\varphi_k(x,\ \xi) = \varphi(x,\ \xi,\ t_k),$$
$$t_i = i\frac{t}{n}.$$

Equation (9) was obtained from (8) by isolating the i-th term in the expression for the sum. Generally speaking, this term of the sum is degenerate owing to the singularity of the kernel of Eq. (7). To be specific, we will construct the i-th term as follows: the integrand function φ is taken for t_i, and the kernel for t_{i-1}. Since the kernel is continuous in the neighborhood of the singularity point, the error obtained is finite and can be made sufficiently small given a suitable choice of the step h.

Equation (9) is a nonhomogeneous Fredholm equation of the second kind which can be solved by simple iteration. The j-th approximation to the solution of this equation is formed from the results of the (j-1)-th approximation according to the following formula:

$$\varphi_i^{(j)}(x) = f_i(x) - \sum_{k=1}^{j-1} \int_l^{j-1} \frac{\varphi_k(x,\ \xi)}{i-k} \exp\left[-\frac{(x-\xi)^2}{h(i-k)}\right] d\xi - \int_l \varphi_i(x,\ \xi) \exp\left[-\frac{(x-\xi)^2}{h}\right] d\xi. \quad (10)$$

An iterative procedure of this type was carried out on a standard type MN analog computer using magnetic drum storage. Thus, as already indicated, in each step we solved Eq. (10) and evaluated the expressions

$$\int_l \varphi_i(x,\ \xi) \exp\left[-\frac{(x-\xi)^2}{h}\right] d\xi,$$
$$\frac{1}{2} \int_l \varphi_i(x,\ \xi) \exp\left[-\frac{(x-\xi)^2}{2h}\right] d\xi,$$
$$\cdots\cdots\cdots$$
$$\frac{1}{n-i-1} \int_l \varphi_i(x,\ \xi) \exp\left[-\frac{(x-\xi)^2}{h(n-i-1)}\right] d\xi, \quad (11)$$

which are used for forming the right sides of the Fredholm equations for the subsequent time layers.

This method makes possible a quite convenient investigation of the nature of the solution for inhomogeneities of various forms (function $f(x, t)$). We examined the two cases

1) $f(x,\ t) = 1 - \exp[-0.3t]$,

2) $f(x,\ t) = \sin t$,

which determine the nonstationary nature of the friction contact $f(x, t) \sim u\tau/j$.

The block diagram of the analog simulation of heat generation at a nonstationary sliding contact (in accordance with Eq. (7)) is shown in Fig. 2.

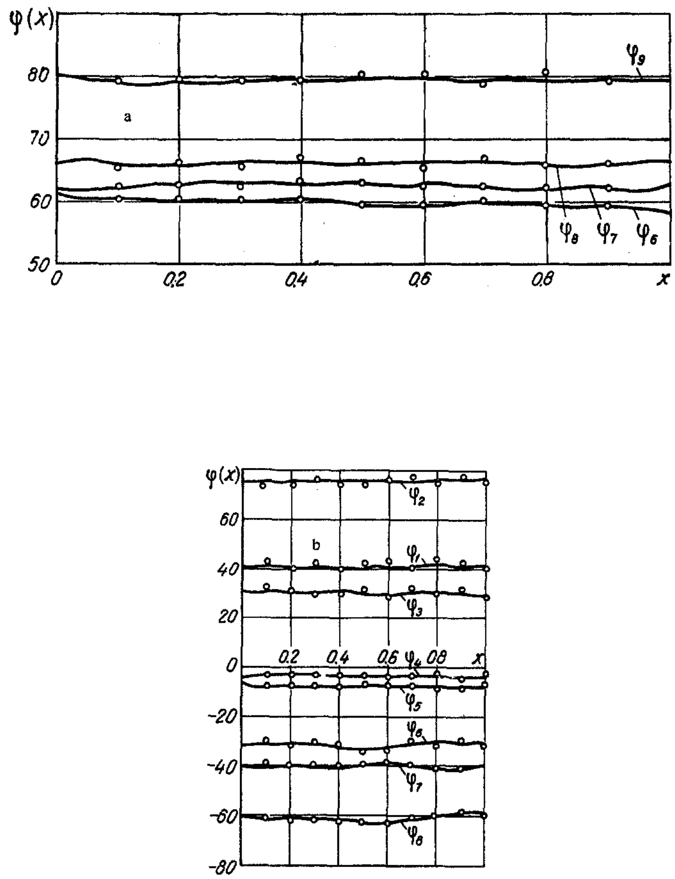

Fig. 3. Graphic representation of solution of thermal problem: a) for the case $f(x, t) = 1 - \exp[-0.3t]$; b) for the case $f(x, t) = \sin t$.

The results of the solution are presented in Fig. 3a for the case $f(x, t) = 1 - \exp[-0.3t]$, and in Fig. 3b for the case $f(x, t) = \sin t$.

NOTATION

λ is the thermal conductivity, kcal/m • hr • deg; c is the specific heat, J/kg • °K; γ is the specific weight, kg/m^3; T is the temperature of the simulated medium, °K; q is the heat flux, kcal/m^2 • hr; u is the relative sliding velocity at a given point of the contact region s, m/sec; τ is the shear stress at the same point; j is the mechanical equivalent of heat, J/cal; a is the thermal diffusivity, m^2/hr; Pe is the Peclet number.

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